

Structural optimization of elliptical-tip star propellant grain using sub-problem approximation

Muhammad Adnan, SUN Bing, ZHANG Jian-wei, Ali Sarosh

(School of Astronautics,

Beijing University of Aeronautics and Astronautics, Beijing 100191, China)

Abstract: A method is presented here for structural optimization of elliptical-tip star grain. The grain structural integrity was improved by minimizing the most critical area of inner bore hoop strain during cool down. Optimization was done by sub-problem approximation method in conjunction with finite element analysis. Both radii of the ellipse were varied during optimization to find the optimal ellipse. The optimization resulted in grain geometry having minimum level of Inner bore hoop strain without violating the preset limits of burning perimeter. The von mises strain at grain inner bore was also reduced in resultant grain.

Key words: solid propellant grain; star grain; viscoelastic analysis; structure optimization; sub-problem approximation

CLC number: V435

Document code: A

Introduction

Star grain geometry has been and still very widely used configuration in solid propulsion systems, because of its simplicity, reliability and neutral burning. Different optimization algorithms have been applied for ballistic optimization of various solid propellant grains. However in order to get required performance it is also important to optimize structural integrity of grain. Grain structural integrity analysis is an evaluation of ability of a solid propellant rocket motor to perform satisfactorily throughout a specified environment^[1]. For this purpose stresses, strains and deformations experienced by grain during its life cycle are analyzed. A distinctive property of solid rocket motor is the manner in which burning surface changes during motor operation. The burning surface at each point recedes in the direction normal to that of the surface resulting in a relation between burning surface and web distance burned depending almost

entirely on initial shape^[2]. This initial shape also depicts structural integrity of grain. In structural integrity analysis, theoretical model was the first choice for researchers because of totally correct and unique solution. However, analytical solution to a viscoelastic problem has only been possible for certain simple configurations and for material properties which are represented by relatively simple viscoelastic models^[3]. Therefore when considering complex shapes and material properties, numerical techniques are necessary for grain structural integrity evaluation.

Typical strain grain has circular tips consisting of one or two circular radii. Changing circular slot tip profile to an elliptical contour has been used previously to maximize peak strain reduction^[4]. Predrag Milos has demonstrated possibilities for designing the star shape propellant grain in such a manner that stresses and strains are minimized without compromising on internal ballistic^[5]. His main idea was to introduce ellip-

Received:2011-03-28; **revision received:**2011-10-12

First Author is Doctoral Candidate of Beijing University of Aeronautics and Astronautics.

E-mail:adnan_1122@yahoo.com

tical fillet instead of classic circular fillet of star valley in order to minimize stress concentration factor. This method involves calculating stress concentration factor by increasing ellipse ratio from 0.3 to 1.0 and choosing one with lowest stress factor.

In this paper two methods were used for structural optimization. First order and sub-problem approximation method integrated with finite element analysis were applied to optimize grain geometry for bore hoop thermal strain. In our problem, ellipse ratio was changed by varying both major and minor radius simultaneously instead of changing only one of the radiuses. This is applied for best possible combination in order to minimize strain value while keeping the burning perimeter in predefined limits.

First order uses gradients of design variables for optimization while sub-problem approximation method replaces the dependant variables by least squares fitting approximation process and converts the problem in an unconstrained one by adding penalty factor. The later method proved to be more efficient while being simple and reliable possessing the ability to find the global optimum in the design space rather than the local minimum.

1 Computation methodology

1.1 Finite element model

Analytically the formulation of a linear elastic static structural problem for a solution by the displacement method is appropriately described by the matrix form

$$[K]\{D\} = \{R\} \quad (1)$$

Where K is the stiffness matrix, D is the unknown and in structural problems it is the nodal displacements and R is the load vector. Solution for unknown D is

$$\{D\} = [K]^{-1}\{R\} \quad (2)$$

For simple case stiffness matrix is obtained by direct method, but generally for most elements a general formula is used, which in its simplest form is

$$[K] = \int [B]^T [E] [B] dV \quad (3)$$

Here B is the strain-displacement matrix, E is the material property matrix or the constitutive matrix and dV is an increment of the element volume V . To take into account the geometrical and material nonlinearities in the finite element analysis, also true for propellant, an incremental approach is often used for numerical simulation. Therefore, E is variable and is updated at each load step to consider the nonlinear material properties. The effect of geometrical nonlinearity is incorporated by modifying the B matrix at each load step.

In order to successfully compute a solution to any boundary value problem the problem must be well posed. The model must include properly defined geometry, material properties, and boundary conditions such that the resulting system is numerically tractable and does not involve singularity. Prime consideration for the structural analysis is to keep desired accuracy while making the model simple^[6].

Since our geometry has natural planes of symmetry so the model was reduced accordingly. In our case the slots occur at regular angular intervals and through the use of displacement type boundary conditions advantage was taken of this symmetry to reduce the model to only a single segment. Likewise for very long structures or structures that are constrained in the “out-of-plane” direction the plane strain assumption is valid which was used to model the grain in only two-dimensional (2-D) geometry. A higher order 2-D, 8-node element having quadratic displacement behavior with plane strain conditions was used. It is well suited to modeling irregular meshes.

1.2 Propellant constitutive model

In order to predict response of viscoelastic propellant, it is necessary to know the parameters of material as function of time and temperature. According to Boltzmann superposition integral, for a simple, non-aging, linear viscoelastic material the stress is related to an arbitrary

strain history through a convolution integral of the form

$$\sigma(t) = \int_0^t E(t - \tau) \frac{d\epsilon}{d\tau} d\tau \quad (4)$$

Where $\sigma(t)$ is stress relaxation and $E(t)$ is relaxation modulus. For temperature dependence it is assumed that the material is described by a thermorheologically simple model. So the temperature dependence is separated from the time dependence through the use of a time-temperature shift and the substitution of a so called “pseudo or reduced time” for real time in the convolution integral. So the integral becomes

$$\sigma(t) = \int_0^t E(\xi - \bar{\xi}) \frac{d\epsilon}{d\bar{\xi}} d\bar{\xi} \quad (5)$$

Here reduced time $\xi=t/a_t$ and a_t is the time-temperature shift factor.

For a complete set of relaxation modulus data a series of constant strain level tests are required at different temperatures within the operating temperature range for the motor. The high temperature short term results are shifted laterally on the time scale and converted into a single master relaxation modulus curve. These shifted values, recorded as a function of temperature, are used to define the shift factor curve. This time temperature shift was modeled using the

Williams-Landel-Ferry (WLF) function.

$$\lg a_t = \frac{C_1(T - T_0)}{C_2 + (T - T_0)} \quad (6)$$

Where C_1 and C_2 are material constants, T_0 is reference temperature and T the test temperature.

The values used here are

$$C_1=5.9548, C_2=161.84, T_0=293.15$$

As the propellant grain is to be modeled using appropriate viscoelastic model, therefore the modulus relaxation data is required to be in a suitable form to be used in structural analysis code. Frequently the relaxation modulus is represented by a Prony series. This is based on Maxwell model having a number of spring-dash-pot Maxwell elements arranged in parallel to each other, while one single spring having stiffness E_∞ present in parallel.

$$E(\xi) = E_\infty + \sum_{i=1}^n E_i \cdot \exp\left(-\frac{\xi}{\lambda_i}\right) \quad (7)$$

Where E_∞ is the equilibrium modulus, E_i are the relaxation coefficients, and λ_i are the relaxation times in the form of reduced time $\lambda_i=t_i/a_t$. In this case, 10 terms Prony series was used. The material properties used are shown in Table 1.

Table 1 Material properties

Property	Propellant	Insulation	Case
Modulus of elasticity/MPa	29.6	59.4	205 000
Coefficient of expansion/ 10^{-6} (1/K)	63	63	6.3
Poisson's ratio	0.49	0.49	0.3

1.3 Grain geometry and loading conditions

The model was built parametrically so that the finite element solver can change parameters as required by optimizer module. The design was programmed in ANSYS parametric design language. The temperature load applied was of 30 K, from initial value of 323 K and time for cool down was 48 h. For simplification it was assumed that due to long cooling time, grain will be uniformly at 293 K after cooling and further-

more temperature will be uniform throughout the grain at any particular time. The initial grain geometry is shown in Fig. 1 and has following parameters, whose values are listed in Table 2.

Number of star points	N
Web fraction	W
Grain outer radius	R
Case outer radius	R_c
Web half angle	φ
Ellipse radii	A and B

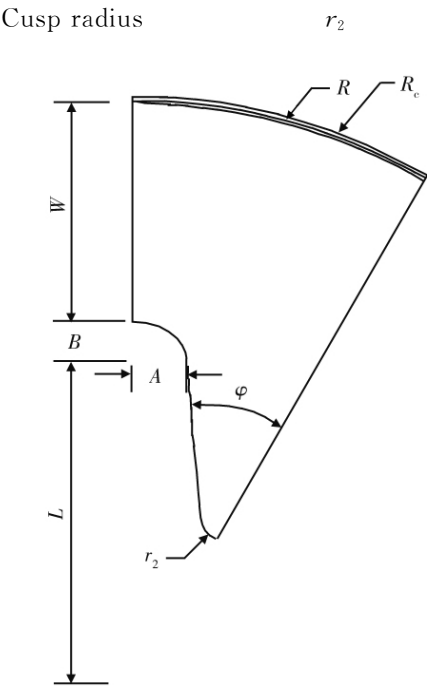


Fig. 1 Grain geometry

Table 2 Design parameters

Parameter name	Parameter value
N	6
Case thickness/mm	4
Insulation thickness/mm	2
Grain radius/mm	594
Web/mm	224
Ellipse major radius A /mm	56
Ellipse minor radius B /mm	40
$L(R-\text{Web}-B)$ /mm	330
Cusp radius/mm	30
Web half angle/($^\circ$)	35

Since both the major and minor axis has to be changed in the optimization and the segment connecting cusp radius to ellipse should always be tangent to both, so x, y coordinates of tangential point at ellipse for each of the different designs is needed. For this purpose analogy of a circle deformed in ellipse was used^[7]. This analogy is shown in Fig. 2. On the left is a circle with tangent line, which is flattened by a factor b/a in the right side.

The triangles in the circle are similar but

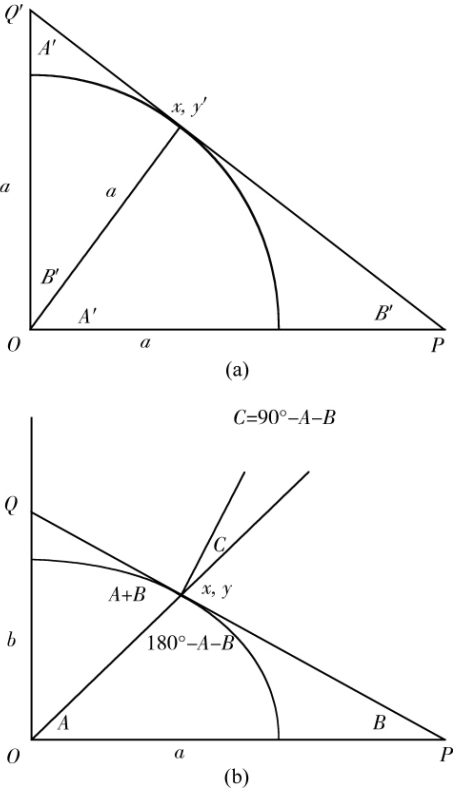


Fig. 2 Ellipse tangential segment

not in the ellipse. Equations (8) – (11) were used to get required values.

$$\tan B' = OQ'/OP \tag{8}$$

$$\tan B = OQ/OP = (b/a)OQ'/OP = (b/a) \tan B' \tag{9}$$

$$\tan A = y/x = (b/a)y'/x = (b/a) \tan A' \tag{10}$$

$$\tan C = \cot(A+B) = \frac{1 - \tan A \tan B}{\tan A + \tan B} \tag{11}$$

The finite element model (FEM) was generated in such a way so as to use the planes of symmetry as depicted in Fig. 3. The boundary conditions applied for symmetry are also evident in it.

1.4 Optimization technique

Initially first order and Sub-problem approximation optimization techniques were employed. The first order method uses gradients of the dependent variables and is computationally expensive. In this method, problem is converted to an unconstrained one by adding penalty function. Each iteration performs gradient calculation to find search direction and line search strat-

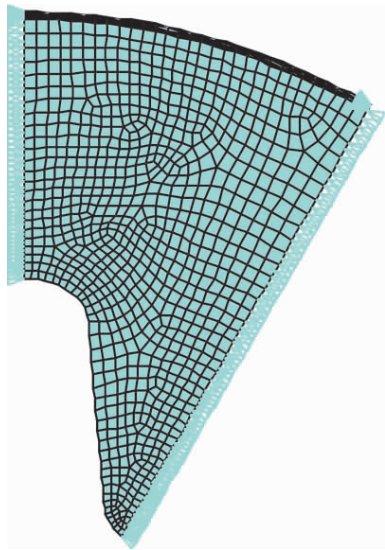


Fig. 3 Finite element model

egy is used to minimize this unconstrained problem. The sub-problem method is a type of zero order optimization which uses only the value of dependent variables during minimization process, unlike first or second order which use first and second derivatives^[8]. The main concepts in this method are the use of approximations for the all dependent variables including objective function, and the conversion of the constrained optimization problem to an unconstrained problem^[9-10]. In this method, the program establishes the relationship between the objective function and the design variables by curve fitting. This is done by calculating the objective function for several sets of design variable values and performing a least squares fit between the data points. The curve or surface obtained is called an approximation. This approximation is then converted to a non constrained problem using penalty function. And finally this approximation is minimized during optimization process until convergence is achieved.

This problem can be stated as

$$\min F = f(x) \tag{12}$$

Subject to

$$g_i(x) \leq \overline{g_i} \quad (i = 1, 2, 3, \dots, m_1) \tag{13}$$

$$h_i \leq h_i(x) \quad (i = 1, 2, 3, \dots, m_2) \tag{14}$$

$$\underline{w_i} \leq w_i(x) \leq \overline{w_i} \quad (i = 1, 2, 3, \dots, m_3) \tag{15}$$

Where f is objective function and g_i, h_i, w_i are state variables containing the design, m_1, m_2, m_3 the number of state variables.

The objective function is approximated as follows

$$\hat{f}(x) = f(x) + E_{\text{error}} \tag{16}$$

Similarly all the state variables are also approximated. The most complex form that the approximations can take on and used here is a fully quadratic representation with cross terms. For the objective function

$$\hat{f} = a_0 + \sum_i^n a_i x_i + \sum_i^n \sum_j^n b_{ij} x_i x_j \tag{17}$$

Where a_i and b_{ij} are coefficients whose values are determined by the weighted least squares technique. Afterwards, this constrained problem is converted into unconstrained one by imposing penalty function.

$$F(x, p_k) = \hat{f} + f_0 p_k \left[\sum_{i=1}^n X(x_i) + \sum_{i=1}^{m_1} G(\hat{g}_i) + \sum_{i=1}^{m_2} H(\hat{h}_1) + \sum_{i=1}^{m_3} W(\hat{w}_i) \right] \tag{18}$$

Here X is the penalty function used to enforce design variable constraints; and G, H , and W are penalty functions for state variable constraints. A sequential unconstrained minimization technique is used to solve this equation.

In our case the objective function is bore hoop strain due to thermal load. This was selected as it is one of the critical area of strain due to thermal loads. The design variables of this problem are A and B while the state variables are ellipse ratio and burning perimeter. All the variables involved and their allowable ranges are given in Table 3.

Table 3 Variable list

Variable	Variable type	Variation range
A/mm	Design	48—64
B/mm	Design	20—60
Ellipse ratio	State	0.26—1.00
Burn perimeter/m	State	0.255—0.275
Bore hoop strain	Objective	Minimize

A set of 30 random designs was generated before optimization to provide initial data points for curve fitting and ease up in convergence of sub-problem and also generating initial feasible sets for first order.

Though optimization is done for reduction in bore hoop strains due to thermal shrinkage, it is also applicable to internal pressurization during ignition. The pressure loading will cause different strain levels but having reduced values. The optimized ellipse has lower strain owing to lower stress concentration factor which is dependent on geometry, not on loading type and same stress concentration is applicable for stress analysis in both types of loadings^[1,11-12].

The design variables and their ranges are in certain relation to grain radius and web because of ballistic requirements of pressure, thrust and sliver during flight. For example $A \approx 0.1R_g$ and $r_2 \approx 0.05R_g$. So with change of grain radius, the values of design variables will change and for small motors the A , B and r_2 will be very small and not practical. Thus extending a dimensional assessment for structural integrity may not be possible.

2 Results and analysis

The resultant grain geometry showed significant reduction in bore hoop strain after optimization. The first order method took more sets to converge compared to sub-problem method but the resulting geometry had higher strain (0.013034) than that of sub-problem (0.012859) method and thus sub-problem method was found to be more efficient. Sub-problem method however involves approximations so it was investigated for errors involved. As the design optimization consists of only 2 design variables having very small design range due to constraints on geometry with a large number of random sets (30), the possibility of unexplored design space is minimal thus the response surface error will also be minimum. To further ensure that errors are at minimal level, a sub-optimization loop based on 40 random sets was later per-

formed. The bore hoop strains further reduced to 0.012813, but the computation time also increased by about 30% because of calculating 10 additional random sets. The reduction in maximum strain value was 18.16% with 30 sets while 18.46% with 40 sets. These results are shown in Table 4.

Table 4 Optimization results

Method	Random sets	Total sets	Hoop strain	Reduction in strain/%
Initial design		1	0.015714	
First order	30	37	0.013034	17.05
Sub-problem	30	33	0.012859	18.16
Sub-problem	40	43	0.012813	18.46

Keeping in view the reduction in strain achieved compared to increase in computation time, it can be deduced that errors in sub-problem method with 30 random sets were at minimum level and results can be used in design process.

The complete results of sub-problem optimization based on 30 sets are listed in Table 5 while the strains of initial and optimized geometry are shown in Fig. 4 – Fig. 7. In addition to reduction in hoop strain, von-mises strain also reduced. The ratio of ellipse in final design thus calculated was 0.47992 with optimum strain level. It must be mentioned here that this optimum ratio changes with change in ratio of grain radius to inner bore radius. So this ratio is suitable for this grain only.

Table 5 Optimization results of sub-problem (30 sets)

Variable	Initial design	Final design
A/mm	56	63.862
B/mm	40	30.649
Ellipse ratio	0.714	0.47992
Burn perimeter/m	0.264833	0.26181
Bore hoop strain	0.015714	0.012859
Von-Misses strain	0.022785	0.019493

The variation of design variables A and B during random sets and approximation is shown in Fig. 8. First set shows initial configuration, sets 2 to 31 are randomly generated sets while 32 and 33 are optimization sets. Bore hoop strain is similarly shown in Fig. 9.

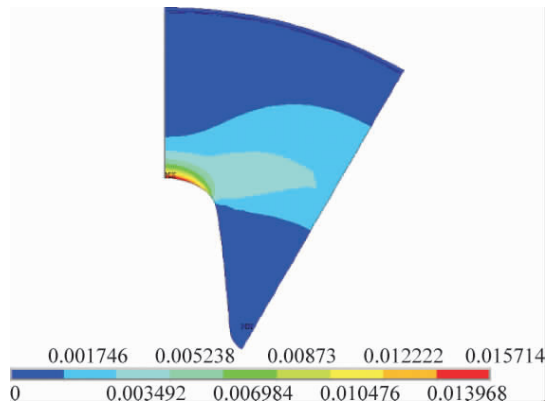


Fig. 4 Initial grain bore hoop strain

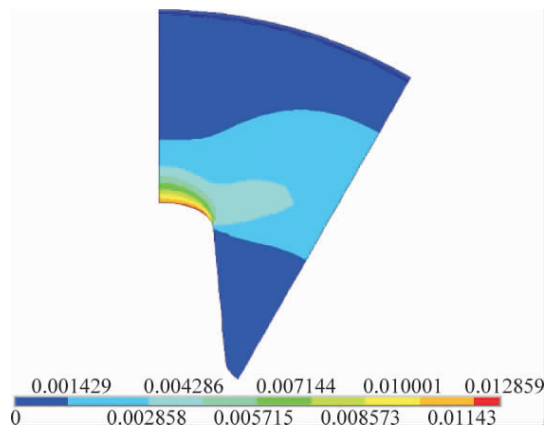


Fig. 5 Optimized grain bore hoop strain

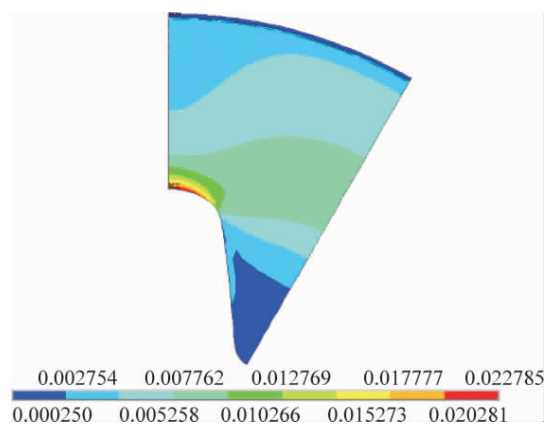


Fig. 6 Initial grain Von-Mises strain

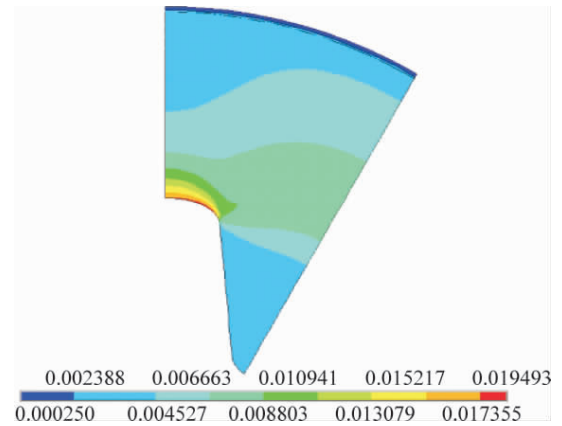


Fig. 7 Optimized grain Von-Mises strain

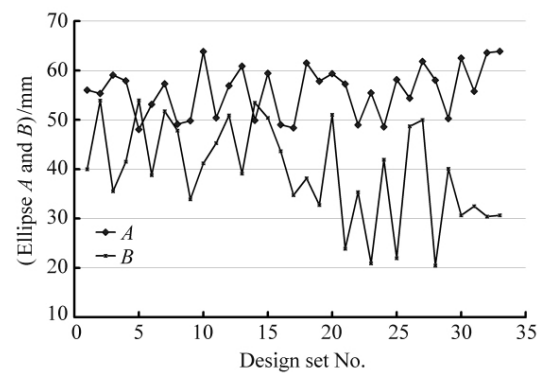


Fig. 8 Variation of ellipse parameters during random and sub-problem optimization

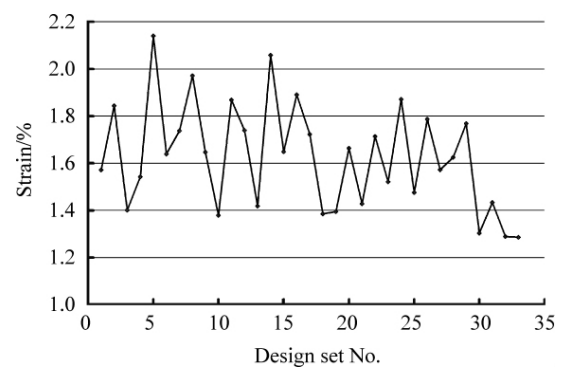


Fig. 9 Variation of bore hoop strain during random and sub-problem optimization

The optimization loop resulted in ellipse having ellipse ratio with optimum value of ellipse minor radius and an attendant increase of ellipse major radius. This is true since bigger the radius, lesser is the strain. Same is observed in results. From the Fig. 8 and Fig. 9 it can be seen that set No. 25 and 28 have low B and high A

values but the strains were not minimal. Furthermore in set No. 27 both *A* and *B* are on higher sides but the strains are still not minimal.

3 Conclusions

In present work, design of star grain on the basis of structural optimization is proposed. Finite element analysis was carried out for structural analysis while first order and sub-problem approximation method were used to minimize the critical strain. Sub-problem method was found more appropriate for star grain problem since significant and comparably higher reduction in bore hoop strain was achieved with lesser computational effort. It was also observed that random sets generated prior to optimization resulted in quick convergence. The approximations involved in this method were observed to be minimal by performing optimization based on additional random sets. Although the optimization performed is for thermal loading but can also be applied effectively for pressure loading. Although this is an approximation method but it has gained significant popularity among developers of structural optimization systems.

Additional work and refinement

Advanced optimization methods can be employed for the further reduction of maximum strains in grain. Moreover it can also be included in ballistic optimization of solid rocket grain so that the final design is optimized not only for ballistic requirements but also for structural integrity considerations.

References:

- [1] Fitzgerald J E. Handbook for the engineering structural analysis of solid propellants[M]. Maryland, USA: Chemical Propulsion Information Agency, (CPIA) Publication 214, Silver Spring, 1971.
- [2] Brooks W T. Solid propellant grain design and internal ballistics[R]. NASA SP-8076, 1972.
- [3] Chyuan S W. Nonlinear thermoviscoelastic analysis of solid propellant grains subjected to temperature loading[J]. Finite Elements in Analysis and Design, 2002, 38 (7): 613-630.
- [4] Chang I S, Adams M J. Three dimensional, adaptive unstructured, mesh generation for solid propellant stress analysis[R]. AIAA 96-3256, 1996.
- [5] Milos P. Geometry optimization of star shaped propellant grain with special attention to minimization of stress and strain[J]. FME Transactions, 2007, 35(1): 35-40.
- [6] Whitehouse A, Dongug P, Mangeston J. Structural assessment of solid propellant grains[R]. France: Neuilly-Sur-Seine, Advisory Group for Aerospace Research & Development, AGARD-AR-350, 1997.
- [7] Dutch S. Mohr circles and conic sections[R/OL]. University of Wisconsin-Green Bay, <http://www.uwgb.edu/dutchs/STRUCTGE/MOHR004.HTM>, 2000.
- [8] Haftka R T. Elements of structural optimization[M]. 3rd ed. Dordrecht, Netherlands: Kluwer Academic Publishers, 1992.
- [9] Chen W, Zhang Y Z, Zhang C J, et al. Thermo-mechanical simulation and parameters optimization for beam blank continuous casting[J]. Materials Science and Engineering: A, 2009, 499(1-2): 58-63.
- [10] ZHANG Jiantao, ZHU Hua, ZHAO Chunsheng. Combined finite element analysis and subproblem approximation method for the design of ultrasonic motors[J]. Sensors and Actuators-A Physical, 2010, 163(2): 510-515.
- [11] Durelli A J, Parks V J. Photoelasticity methods to determine stresses in propellant-grain models[J]. Experimental Mechanics, 1965, 5(2): 33-45.
- [12] Fournery M E, Paramerter R R. Parametric study of rocket grain configurations by photoelastic analysis[R]. AFSC Report No. AFRPL-TR-66-52, 1966.
- [1] Fitzgerald J E. Handbook for the engineering structural a-